Study of the Optimal Switching Levels of Adaptive Modulation for a Two-user System under Constant Power Condition

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Abstract—In this paper we analyze the problem of optimizing the switching levels of adaptive modulation for a two-user system under constant power condition, in order to maximize the overall system throughput for a target average bit-error-rate (BER) for each user. Since the optimal switching levels depend on the probability density of the signal-to-interference-noise ratio (SINR), expression for this probability density is derived and the optimal switch levels are obtained through a Lagrange optimization technique. This approach can be readily extended to multiple-user case. It is shown by simulation that the proposed jointly optimizing method achieves the average BER target for each user while an individually optimizing method fails. The effects of these two methods and of the interference coefficients on the average system throughput are also analyzed.

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Wireless communication channels typically exhibit channel quality fluctuations, and to mitigate their detrimental effects a number of approaches have been proposed in the literature. Methods based on adaptive modulation according to the near-instantaneous channel quality information have gained popularity. For Rayleigh fading channel, Chung and Goldsmith [1] showed that variable power variable-rate schemes are optimal, but they also found that the extra throughput achieved by the additional variable power assisted adaptation over the constant power variable rate schemes is marginal for most types of fading channels.

Constant power variable rate adaptive scheme is also analyzed by Choi and Hanzo [2], where the target average BER forms the constraint condition. They established the relationship among the optimal mode switching levels, and found that although such relationship holds regardless of the underlying channel scenarios, the switching levels do depend on the statistics of the channel quality.

The above studies were focused on single user scenario. Yet in wireless communication, a user typically suffers from the interference of other users, which in the GSM system may be due to multipath channel conditions, and in the CDMA system may stem from the loss of orthogonality among the spread sequences. Several approaches have been proposed based on power control and base station selection [3, 4], with little consideration for adaptive modulation. In [5] Qiu and Chawla approached the multiuser problem by using power control and adaptive modulation. They showed that adaptive modulation without power control scheme outperforms SINR-balance power control scheme in terms of overall throughput, a finding that indicates the significance of constant power adaptive modulation for multiuser system. In their approach, the switching levels were obtained by using the method due to Webb and Steele, and a parametric approximation for throughput for all modulation modes was also adopted.

In this paper we propose a constant-power adaptive modulation scheme for downlink transmission in a multiuser system. It is based on a Lagrangian optimization technique. We consider two-user case here to highlight the concept. Extension to multiuser case can be readily carried out.

The downlink transmission model is shown in Fig. 1, which is similar to the one used in [6]. The received signals are for user 1

\[ r_1 = \sqrt{s_1 h_1 d_1} + \sqrt{\beta s_2 h_2 d_2} + n_1 \]  

(1)

and for user 2

\[ r_2 = \sqrt{s_2 h_2 d_2} + \sqrt{\alpha s_1 h_1 d_1} + n_2 \]  

(2)

where for user \(ks_k\) are transmit power, \(h_k\) are the channel coefficients, \(d_k\) are transmitted symbols, and \(\alpha\) and \(\beta\) are the interference coefficients from user 2 to user 1 and from user 1 to user 2,
respectively. The received signal-to-interference-noise ratios are for user 1

\[ \gamma = \frac{s_1 h_1}{\beta s_2 h_2 + \sigma^2_{n_1}} \]  

and for user 2

\[ \varsigma = \frac{s_2 h_2}{\alpha s_1 h_1 + \sigma^2_{n_2}} \]  

To comply by the practical system, here we consider the case where the modulation modes for each user are no-transmission, BPSK, QPSK, 16-QAM, and 64-QAM. Other cases corresponding to alternative set of modulation modes can be analyzed similarly. Let the associated switching levels be \{\gamma_0, \gamma_1, \gamma_2, \gamma_3\} for user 1 and \{\varsigma_0, \varsigma_1, \varsigma_2, \varsigma_3\} for user 2. The overall throughput for the 2-user system is

\[ B_{\text{total}} = \sum_{i=0}^{3} b_i \int_{\gamma_i}^{\gamma_{i+1}} f_\gamma(u) du + \sum_{k=0}^{K-1} b_k \int_{\varsigma_k}^{\varsigma_{k+1}} f_\varsigma(v) dv, \]  

where \( f_\gamma(u) \) and \( f_\varsigma(v) \) are the probability densities of user 1 and 2 respectively, \( b_k \) is the BPS throughput under modulation mode \( k \). For user 1, the average BER is

\[ \overline{\text{BER}}_1 = \frac{\sum_{i=0}^{3} b_i \int_{\gamma_i}^{\gamma_{i+1}} \text{BER}_i(u) f_\gamma(u) du}{\sum_{i=0}^{3} b_i \int_{\gamma_i}^{\gamma_{i+1}} f_\gamma(u) du}, \]  

where the probability density \( f_\gamma(u) \) is

\[ f_\gamma(u) = \frac{1}{4\sigma_1^2 \sigma_2^2 s_1} e^{-\frac{2\gamma_{\text{sys}}}{\sigma_1^2 \sigma_2^2 s_1}} \left[ \frac{4\sigma_1^2 \sigma_2^2 s_1 \gamma_{\text{sys}}}{2\sigma_2^2 \beta s_2 u + 2\sigma_1^2 s_1} + \frac{(4\sigma_1^2 \sigma_2^2 s_1 \beta s_2)^2}{(2\sigma_2^2 \beta s_2 u + 2\sigma_1^2 s_1)^2} \right]. \]  

The average BER and the probability density \( f_\varsigma(v) \) for user 2 can be expressed similarly.

The objective here is to optimize the overall throughput while maintaining the average BER of each user at the specified level \( \overline{\text{BER}}_1 \) and \( \overline{\text{BER}}_2 \) respectively. It is an optimization problem with constraints, whose Langrange equation is

\[ J(\gamma_0, \gamma_1, \ldots, \gamma_{N-1}, \varsigma_0, \varsigma_1, \ldots, \varsigma_{K-1}) = \sum_{i=0}^{N-1} b_i \int_{\gamma_i}^{\gamma_{i+1}} f_\gamma(u) du + \sum_{k=0}^{K-1} b_k \int_{\varsigma_k}^{\varsigma_{k+1}} f_\varsigma(v) dv + \lambda_{11} \sum_{i=0}^{N-1} b_i \int_{\gamma_i}^{\gamma_{i+1}} (\text{BER}_i(u) - \overline{\text{BER}}_1) f_\gamma(u) du \]

\[ + \lambda_{12} \sum_{k=0}^{K-1} b_k \int_{\varsigma_k}^{\varsigma_{k+1}} (\text{BER}_k(v) - \overline{\text{BER}}_2) f_\varsigma(v) dv, \]  

where

\[ f_\gamma(u) = \frac{1}{4\sigma_1^2 \sigma_2^2 s_1} e^{-\frac{2\gamma_{\text{sys}}}{\sigma_1^2 \sigma_2^2 s_1}} \left[ \frac{4\sigma_1^2 \sigma_2^2 s_1 \gamma_{\text{sys}}}{2\sigma_2^2 \beta s_2 u + 2\sigma_1^2 s_1} + \frac{(4\sigma_1^2 \sigma_2^2 s_1 \beta s_2)^2}{(2\sigma_2^2 \beta s_2 u + 2\sigma_1^2 s_1)^2} \right]. \]
where $\lambda_{11}$ and $\lambda_{12}$ are the Langrange parameters. Taking derivatives with respect to $\gamma_j$ and $\varsigma_j$ and forcing them to be zero give rise to the following equations

$$\frac{1}{b_i - b_{i-1}} (b_i BER_i(\gamma_i) - b_{i-1} BER_{i-1}(\gamma_i)) = \frac{BER_1}{\lambda_{11}}$$

and

$$\frac{1}{b_k - b_{k-1}} (b_k BER_k(\varsigma_k) - b_{k-1} BER_{k-1}(\varsigma_k)) = \frac{BER_2}{\lambda_{12}}$$

It is seen that (9) and (10) are decoupled equations. The procedure for solving each of them is well documented in [2]. The key results from that study are the following: 1) the optimal values of $\gamma_j$ depend on that of the first switch level $\gamma_0$; 2) The optimal $\gamma_0^*$ depends on the probability density $f_\gamma(u)$. Similar results apply to $\varsigma_j$. Determination of $\gamma_0^*$ is through solving the constraint equation

$$\sum_{i=0}^{N} b_i \int_{\gamma_i}^{\gamma_{i+1}} BER_i(u) f_\gamma(u) du - \frac{BER_1}{\lambda_{11}} \sum_{i=0}^{N} b_i \int_{\gamma_i}^{\gamma_{i+1}} f_\gamma(u) du = 0$$

which is a highly nonlinear equation which requires a numerical routine in general.

In the following we present several numerical simulation cases to illustrate the effects of several parameters on the system BER and throughput performance. Fig. 2 compares the BER performances of joint optimization and separate optimization. The interference coefficients are $\alpha = \beta = 0.01$. The target average BERs for both users are $10^{-4}$, and the noise variances are $-20$ dBm. It is seen that the BER under joint optimization attains the target average BER, while the BER under separate optimization is larger than the target average BER, with a rapidly increasing tendency as the SNR of each user increases. This is because higher SNR for one user also means higher interference to the other user, which will present a problem when separate optimization is used. Fig. 3 compares the average throughput performances of joint optimization and separate optimization. The average throughput of separate optimization is slightly better than that of joint optimization, but from above we know that the average BER of separate optimization does not fulfill the BER requirements. Fig. 4 illustrates the effect of interference coefficients on the average throughput using joint optimization. The interference coefficients are symmetric ($\alpha = \beta$, ranging from 0.001 to 1. The average SNR for user 1 is 20 dB, while that of user 2 ranges from 0 to 40 dB. The average throughput shows a convex behavior for $\alpha = 0.1$ and 1, and a concave behavior for $\alpha = 0.01$ and 0.001. This is expected since for high values of interference coefficient, when the SNR of user 2 begins to increase from 0 dB, loss of throughput of user 1 due to high interference outweighs gain of throughput of user 2, and this trend continues until the SNR of user 2 approaches 20 dB, after which, the gain of throughput of user 2 outweighs loss of throughput of user 1. The same analysis applies to small values of interference coefficient.
Figure 4: The impact of interference coefficients on system throughput using joint optimization.

REFERENCES