Advanced GILD EM Modeling and Inversion

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Abstract

In this paper, we propose a new Advanced Global Integral and Local Differential (GILD) Electromagnetic (EM) modeling and inversion (AGILD). We derive a new EM differential integral strip equation. We used it on the boundary strip and MAXWELL differential equation in the internal domain to construct the AGILD. A new 9-cell local weak regularizing inversion scheme is presented. The new AGILD is a fast modeling and reasonable stable inversion. AGILD preserved GILD’s merits and is simpler than the GILD. Recently, we developed a novel and powerful GL modeling and inversion. The GL and AGILD can jointly work in the geophysics, GPR and weather radar imaging, optical electric, nondestructive testing, material sciences, nanometer materials etc. sciences and engineering.

1. Introduction

The electromagnetic inversion is strongly dependent on the frequency. In the high frequency, the EM inversion is wave field inversion with weak ill posed. In the low frequency, the EM inversion is diffusion inversion with strong ill posed. In the medical field, the EM inversion is mixed ill posed. In general, Geophysical EM inversion in low frequency is ill posed. The EM inversion in material science, sensor, radar, and nondestructive is weak or medial ill posed. Therefore, high resolution and ill posed conflict in the EM inversion. That is a main difficulty for the EM inversion. Xie and Li proposed Global Integral and Local Differential GILD Modeling and Inversion in 1997. The GILD EM modeling and nonlinear inversion has been published in Xie and Li’s papers in SEG in 1997, 3D Electromagnetcs in 1999[5], Physica D in 1999 [4], and Geophysics [6] in 2000. GILD EM modeling and inversion overcame main difficult in EM inversion. In this paper, we proposed an advanced GILD EM modeling and inversion. We derive a EM differential integral boundary strip equation. We use it on the boundary strip and MAXWELL differential equation in the internal domain for the AGILD. A new FEM EM integral differential equation on the boundary strip and the Galerkin FEM EM equations in the internal domain to construct the AGILD EM modeling and inversion. A new 9-cell local weak regularizing inversion scheme is presented. The AGILD is a fast modeling and a reasonable stable inversion. AGILD has GILD’s merits and is simpler than the GILD. Recently, we developed a novel and powerful GL modeling and inversion. The GL and AGILD can jointly work in the geophysics, GPR and weather radar imaging, optical electric, nondestructive testing, material sciences and engineering. The arrangement of this paper is as follows. In section 2, we propose the new magnetic field differential integral equation on the boundary strip. The new advanced GILD EM modeling is described the section 3. In the section 4, we present the new advanced GILD EM inversion. The simulation and applications are described in the section 5. Finally, we describe discussion and conclusion in the section 6.

2. A New Magnetic Field Differential Integral Equation on the Boundary Strip

In this section, we proposed a new magnetic field differential integral equation on the boundary strip \( \Omega_s \).

\[
H(r) = H_{bo}(r) + \int_{\partial \Omega_s} \frac{1}{\sigma + i \omega \epsilon} \left( G'_b \times \nabla \times H \right) \cdot dS - \int_{\partial \Omega_b} \frac{1}{\sigma + i \omega \epsilon} \left( \nabla \times G_b^M \times H \right) \cdot dS \\
+ \int_{\Omega_s} \frac{(\sigma + i \omega \epsilon) - (\sigma_b + i \omega \epsilon_b)}{\sigma + i \omega \epsilon} E_b^M \cdot (\nabla \times H) \, dr'
\]

(1)

where \( \Omega_s \) is a strip domain located between the double boundary layers \( \partial \Omega_0 \cup \partial \Omega_b \), and with \((\sigma - \sigma_b) + i \omega (\epsilon - \epsilon_b) \neq 0\). \( \partial \Omega_b \) is the external boundary of the strip domain \( \Omega_b \), \( \partial \Omega_0 \) is the internal boundary of the strip domain \( \Omega_s \), \( H(r) \) is the unknown magnetic field, \( H_{bo}(r) \) is incident magnetic field, \( G_b^M(r', r) \) is the magnetic Green function excited by a magnetic dipole source at point \( r \), \( \sigma \) is the conductivity, \( \epsilon \) is the dielectric, \( \epsilon_b \) is the background conductivity, \( \sigma_b \) is the background dielectric, \( \mu \) is the permeability, \( \omega \) is the angle frequency, \( \nabla_{r'} \) is curl operator with respect to \( r' \). The unknown magnetic field \( H(r) \) in (1) is continuous when \( \mu \) is continuous or constant. The equation (1) is second type differential integral equation with weak singularity and convergent Green kernel.
3. The Advanced GILD EM Modeling

3.1 Finite Element Equation of The Equation (1) on The Boundary Strip

The strip domain $\Omega_s$ is inside the double layered boundary. The $\Omega_s$ is divided into a strip set $\Omega_s^b$ of the cubic elements with 8 nodes. The point $r$ is located in external layer, $r'$ is located in the internal layer $\partial \Omega_i$ and $\Omega_s^b$. The $\partial \Omega_i$ is divided into a set $\partial \Omega_i^b$ of the rectangle elements which is the internal layer of the $\Omega_s^b$. The differential integral equation (1) is discretized as

$$
H^h(r_i) = H^h_{bd}(r_i) + \int_{\partial \Omega_i^b} \left( \frac{1}{\sigma + i \omega \varepsilon} G^M_b(r', r_i) \times \nabla \times H^h(r') \right) \cdot dS
$$

$$
- \int_{\partial \Omega_i^b} \frac{1}{\sigma_b + i \omega \varepsilon_b} (\nabla \times G^M_b(r', r_i) \times H^h(r')) \cdot dS
$$

$$
+ \int_{\Omega_s^b} \frac{(\sigma + i \omega \varepsilon) - (\sigma_b + i \omega \varepsilon_b)}{\sigma + i \omega \varepsilon} E^M_b \cdot (\nabla \times H^h) dr'
$$

(2)

$H^h(r_i)$ is the finite element approximation of $H(r)$, The equation (2) is collocation finite element equation of the (1). The equation (2) is the matrix equation

$$
K_{bb} H^h_b + K_{bd} H^h_d = Q^h_b
$$

(3)

where $H^h_b$ is the magnetic field on the finite element external boundary $\partial \Omega_i^b$, $H^h_d$ is the magnetic field on the finite element internal boundary $\partial \Omega_i^b$, $K_{bb}$ is the finite element matrix on external boundary $\partial \Omega_i^b$ and relative to $\partial \Omega_i^b$, $K_{bb}$ is the finite element matrix on external boundary $\partial \Omega_i^b$ and relative to $\partial \Omega_i^b$, $K_{bb}$ and $K_{bd}$ are boundary full matrix.

3.2 Finite Element Equation in the Internal Domain

In the internal domain $D$ which is inside $\partial \Omega_i$, we use Galerkin FEM method \[9-10\] and the cubic element to discretize the MAXWELL magnetic field differential equation

$$
\nabla \left( \frac{1}{\sigma + i \omega \varepsilon} \nabla \times H \right) + i \omega \mu H = Ms
$$

(4)

The Galerkin finite element equation \[9-10\] is as follows

$$
\int_{\partial \Omega_i^h} \frac{1}{\sigma + i \omega \varepsilon} (\nabla \times H^h \times (\phi_i \vec{e}) \cdot dS + \int_{\partial \Omega_i^h} \frac{1}{\sigma + i \omega \varepsilon} \nabla \times H^h \cdot \nabla \times (\phi_i \vec{e}) dr
$$

$$
+ i \omega \mu \int_{\Omega_s^b} H^h \cdot (\phi_i \vec{e}) dr = \int_{\Omega_s^b} M_s \cdot (\phi_i \vec{e}) dr
$$

(5)

where $\Omega_s^b$ is the finite element domain of the internal domain $D$, $\partial \Omega_i^h$ is finite element set of the $\partial \Omega_i$, the $\phi_i$ is the finite element base function, the $\vec{e}$ is unit vector. The equation (5) is the matrix equation

$$
K_{bb} H^h_b + K_{bd} H^h_d = Q^h_d
$$

(6)

where $H^h_b$ is the magnetic field on the finite element external boundary $\partial \Omega_i^b$, $H^h_d$ is the magnetic field inside the finite element internal domain $\Omega_s^b$, $K_{bb}$ is the finite element matrix in internal domain $\Omega_s^b$ and relative to $\partial \Omega_i^b$, $K_{bb}$ is the finite element matrix in internal domain $\Omega_s^b$ and relative to $\partial \Omega_i^b$, $K_{bb}$ and $K_{bd}$ are sparse matrices.

3.3 The Advanced GILD EM Modeling

The collocation finite element matrix equation (3) and the Galerkin finite element matrix equation (6) are coupled to construct the advanced GILD EM modeling.

The GILD EM modeling algorithm from the internal domain to the boundary is performed, in parallel.

4. Advanced GILD EM Inversion

The GILD EM nonlinear inversion was proposed by us in 1997. The papers was published in SEG proc in 1997, 1999[5], 2000[6]. We present the advanced GILD EM inversion here.

4.1 First Type Differential Integral Equation for EM Parameters
We proposed a first type differential integral equation for EM conductivity and dielectric parameters. The equation is used to connect the EM parameters on the boundary strip to the measured magnetic field data in the data site.

\[
H_D(r_d) = H_{b_0}(r_d) + \int_{\partial \Omega^h_b} \left( \frac{1}{\sigma^h + i \omega \varepsilon^h} (G^M_b(r', r_d) \times \nabla \times H^h(r')) \cdot dS \right) - \int_{\partial \Omega^h_b} \left( \frac{1}{\sigma_b + i \omega \varepsilon_b} (\nabla \times G^M_b(r', r_d) \times H^h(r')) \cdot dS \right) + \int_{\Omega^h_s} \frac{(\sigma^h + i \omega \varepsilon^h) - (\sigma_b + i \omega \varepsilon_b)}{\sigma^h + i \omega \varepsilon^h} E^M_b(r', r_d) \cdot (\nabla \times H^h(r')) dr' \tag{7}
\]

The equation (7) is similar to equation (2), however, they are totally different in mathematical physical meaning. The equation (7) is the first type nonlinear differential integral equation with respect to unknown conductivity \(\sigma^h\) and dielectric parameter \(\varepsilon^h\). The equation (2) is the second linear differential integral equation with unknown magnetic filed \(H^f\). In the equation (7) \(H^h\) is also unknown magnetic field, moreover, it is a implicit nonlinear operator of the \(\sigma^h\) and \(\varepsilon^h\). So, the equation (7) is nonlinear. The \(r_d\) is data location, \(H_D(r_d)\) is the magnetic field data in the data location \(r_d\). \(H_D(r_d) = H(r_d, r_s, \omega f), d=1, \ldots nd, s=1, \ldots ns, f=1, \ldots nf\). Let \(\alpha^h_s\) is finite element vector \(\sigma^h\) in the strip with \(ms\) components. \(\varepsilon^h_i\) is finite element vector \(\varepsilon^h\) on the internal boundary strip with \(mi\) components. \(\varepsilon^h_i\) is finite element vector \(\varepsilon^h\) on the internal boundary strip with \(mi\) components. The equation (7) includes \(nd \times ns \times nf\) nonlinear equations. After regularizing with regularizing parameter \(\alpha_s\) and linearization, the linearized equation system of the (7) with respect to unknown parameters \(\delta\alpha^h_s, \delta\varepsilon^h_i, \delta\sigma^h, \delta\varepsilon^h\) will be the following matrix equation

\[
\varsigma_{ss} \left[ \begin{array}{c} \delta\alpha^h_s \\ \delta\varepsilon^h_i \end{array} \right] + \varsigma_{s} \left[ \begin{array}{c} \delta\alpha^h_s \\ \delta\varepsilon^h_i \end{array} \right] = \delta H_{Ds} \tag{8}
\]

where \(\varsigma_{ss}\) is only \(2ms \times 2ms\) strip full matrix on the strip cells relative to the strip cells. \(\varsigma_{si}\) is only \(2ms \times 2mi\) strip full matrix on the \(2ms\) strip cells relative to the \(2mi\) internal strip cells. The internal strip is belonging to the internal domain.

### 4.2 Galerkin Equation for EM Parameters

The Galerkin equation (5) is a linear equation for finite element magnetic field \(H^h\), when conductivity \(\sigma\) and dielectric parameter \(\varepsilon\) are unknown. In the EM parameter inversion, the conductivity \(\sigma\) and dielectric parameter \(\varepsilon\) are unknown, the Galerkin equation (5) is a nonlinear equation for the \(\sigma^h\) and \(\varepsilon^h\). The \(H^h\) is also unknown magnetic field, moreover, the \(H^h\) is the implicit nonlinear operator of the \(\sigma^h\) and \(\varepsilon^h\).

\[
\int_{\partial \Omega^h_D} \frac{1}{\sigma^h + i \omega \varepsilon^h} \nabla \times H^h \times (\phi_i \vec{e}) dS + \int_{\Omega^h_D} \frac{1}{\sigma^h + i \omega \varepsilon^h} \nabla \times H^h \cdot \nabla \times (\phi_i \vec{e}) dr + i \omega \mu \int_{\Omega^h_D} H^h \cdot (\phi_i \vec{e}) dr = \int_{\Omega^h_D} M_s \cdot (\phi_i \vec{e}) dr \tag{9}
\]

After linearization and weakly regularizing of the (9) with weak regularizing parameter \(\beta\), the weak regularizing linearized equation of (9) is the following matrix equation

\[
\varsigma_{ss} \left[ \begin{array}{c} \delta\alpha^h_s \\ \delta\varepsilon^h_i \end{array} \right] + \varsigma_{s} \left[ \begin{array}{c} \delta\alpha^h_s \\ \delta\varepsilon^h_i \end{array} \right] = \delta H_{Ds} \tag{10}
\]

\([\delta\alpha^h_s \ \delta\varepsilon^h_i]\) is the increment vector of \(\delta\alpha^h\) and \(\delta\varepsilon^h\) in the cells of the internal domain \(D\). The \([\delta\alpha^h_s \ \delta\varepsilon^h_i]\) is a part of \([\delta\alpha^h_s \ \delta\varepsilon^h_i]\). \(\varsigma_{ss}\) is the matrix on the cells of the internal domain \(\Omega^h_s\) and relative to the boundary strip cells \(\Omega^h_d\). \(\varsigma_{s}\) is the matrix on the cells of the internal domain \(\Omega^h_s\) and relative to the \(\Omega^h_d\) with \(\partial \Omega^h_d \subset \Omega^h_d\). \(\varsigma_{ss}\) and \(\varsigma_{s}\) are sparse matrices.

### 4.3 Advanced GILD EM Inversion

The regularizing linearization finite element equation (8) on the strip \(\Omega^h_s\) and the weak regularizing linearization finite element equation (10) on the strip \(\Omega^h_d\) are coupled to construct the advanced GILD EM Inversion. The AGILD EM inversion algorithm from the internal domain to the boundary strip is performed, in parallel.
4.4 Regularizing
Because the low frequency EM parameter inversion is ill posed, the regularizing method is necessary to make the inversion stable. We used the Tikhonov regularizing for first type EM parameter differential integral equation (7) on the boundary strip.

The weak regularizing is used for the Galerkin parameter equation (9). The strong and weak regularizing methods are consistently coupled to make AGILD EM inversion stable and high resolution.

4.5 2.5D AGILD EM Modeling and Inversion
In the electromagnetic engineering and sciences, the 2.5D problem with 3D EM field and 2D EM parameters is an important topic. Suppose that the electric conductivity $\sigma$ and dielectric parameter $\varepsilon$ are invariable on the variable $y$, i.e., the $\sigma(x, z)$ and $\varepsilon(x, z)$ are only two dimensional functions. Upon substituting Fourier representation for $H_h(r)$ into the differential integral equation (2) and Galerkin equation (5), we obtain the 2.5D advanced AGILD EM modeling. To substitute Fourier representation for $H^b_h(r)$ into the (7) and (9), we obtain the 2.5D AGILD EM inversion[7].

5. The Simulation and Application

5.1 AGILD EM Modeling Simulation
We used many models to test our advanced GILD modeling code. The AGILD simulations of the bone models are presented in this section. Figure 1 shows the back bone model, which consists of a cubic domain {−25, 65 \mu m; −40, 40 \mu m; 0, 65 \mu m} with background resistivity $\rho=20$ ohm-m, background permittivity $\varepsilon_b = 1000\varepsilon_0$. The mesh is 90 x 80 x 85. In the back bone simulation model, resistivity $\rho = 20$ ohm-m, permittivity $\varepsilon = 5000\varepsilon_0$, $\varepsilon_0 = 8.854 \times 10^{-12} F/m$. A vertical magnetic dipole source is at (0, 0, 40) to excite the EM field. By AGILD EM modeling calculation, the vertical magnetic field of the model 1 in the plane, $y=0$, are plotted in Figure 3. The tests show that the AGILD EM modeling algorithm and software are fast and accurate.

![Figure 1: The BackBone Model](image1)

![Figure 2: Bone Image Using AGILD Inversion](image2)

![Figure 3: The magnetix field Hz of backbone model](image3)

5.2 AGILD EM Inversion Simulation
We received vertical magnetic field $Hz$ in the 40 data sites on the surface $z=0$ for the model excited by four sources. We used the background EM parameters as starting model to simulate AGILD inversion. After 25 times AGILD inversion iterations, the inversion image by AGILD inversion is obtained. The figure 2 shows the born image using the AGILD inversion. The high resolution images are obtained by synthetic data.

5.3 Applications
The AGILD modeling and inversion has wide applications in the material sciences, nondestructive testing, earthquake exploration, geophysics exploration, and environment engineering and civil engineering.[4-10]. Paper [9] is the first 3D FEM journal paper in China that show that author developed 3D finite element method first in China. In this paper, author proposed a higher convergent that displacement and stress have convergent rate $O(h^4)$ in [9] that is superconvergence.

6. Conclusion and Discussion

6.1 Frequency and Data
Many synthetic and field data simulations show that the advanced GILD modeling is fast and accurate, the advanced GILD inversion has high and reasonable resolution for middle and high frequency.

The inversion imaging resolution depends on the scattering data quality, frequency, configuration, and target
size. The AGILD inversion has ability to invert field data with 20% ∼ 30% noises.

6.2 Merits of AGILD Method

We proposed boundary strip differential integral equation to simplify and upgrade GILD method. AGILD has same merits as GILD method in paper [4-6,8]. The strip differential integral equation is used to be exact boundary condition that reduced boundary reflect error. The Galerkin equation for inversion that is lower ill posed sparse matrix. The strip differential integral equation generate double layers finite element matrix equation (3) for modeling and generate double layers finite element matrix equation (8) for inversion. These main upgrades speed and simplify GILD parallel algorithm and preserve GILD’s accurately and high resolution.

6.3 GL Method and AGILD Inversion

Recently we develop a new novel Global and Local electromagnetic field modeling and inversion[1-3,8]. We proposed an explicit nonlinear formulation for the GL EM inversion. The GL EM modeling and inversion are alternative processes. The advantages of GL method is that, (1) there is no any big matrix need to solve, (2) The complex artificial absorption conditions are removed, (3) Combining analytic and numerical method. The GL method is totally different from FEM, FD, Born like method. GL method and AGILG can be perfectly join for inversion.

REFERENCES