Anomalous Selective Transparency of Many-Mode Surface-Corrugated Waveguides

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Abstract

We study the role of long-range correlations in random surface profiles for the transport through many-mode waveguides. It is found that the correlations that result in a perfect transmission for a single-mode case, lead to a quite unexpected phenomenon when the number of modes is large. Specifically, we show that these long-range correlations give rise to the appearance of non-interacting and completely transparent 1D channels, the number of which is governed by the correlation parameter and can be as large as the number of propagating modes. As a result, the total transmission of waveguides can be significantly enhanced in comparison with uncorrelated surface profiles. Moreover, the waveguide can be fully transparent. This effect is directly related to the fact that for many-mode surface scattering there are many scattering lengths and the famous one-parameter scaling known to be held for bulk scattering, in our case is not valid. Therefore, the transport through different channels can be separated by a proper choice of long-range correlations, and transition from ballistic to diffusive or localized regime is expected to be sharp enough in order to observe them experimentally.

1. Introduction

During last few years there is a burst of interest to the transport problem of one-dimensional systems with correlated disorder. This fact is due to intriguing results that may have applications both to electromagnetic and electron/optic devices. In particular, it was shown [1-3] that any desired combination of transparent and nontransparent frequency windows can be observed by an appropriate choice of random potentials with specific long-range correlations. Experimental realization [4] of such potentials in single-mode waveguides with inserting delta-like scatters, has confirmed the theoretical predictions.

The study of wave propagation through surface-disordered waveguides is important for applications in optics fibers, remote sensing, radio wave propagation, shallow water waves, random photonic lattices, etc. Also, the waveguiding propagation describes the electronic transport in mesoscopic conducting structures. For this reason, based on the theory of Anderson localization, we have recently analyzed anomalous surface-controlled transmission in waveguides with one open channel [5]. It was shown analytically and by direct numerical simulations that single-mode waveguides with a predefined transparency can be fabricated by a respective construction of long-range-correlated random surface profiles.

In this contribution we briefly discuss our results concerning the effect of long-range correlations in rough surfaces on selective transport of many-mode plane waveguides (or quasi-1D electron wires).

2. Waveguides with surface disorder

In the case of many-mode waveguide with random rough surface, the total average transmittance \( <T> \) can be expressed as a sum of partial transmittances \( T_n \) for every \( n \)-th propagating mode,

\[
<T> = \sum_{n=1}^{N_d} T_n .
\]

(2.1)

Here \( N_d = \lfloor kd / \pi \rfloor \) is the total number of propagating modes (conducting channels) with \( \lfloor \ldots \rfloor \) as an integer part, and \( d \) is the average waveguide width. The wave number \( k \) is equal to \( \omega / c \) for a classical wave of frequency \( \omega \), and to the Fermi wave number for electrons.
From the general theory of surface scattering [6] it follows that the transmission of any \( n \)-th conducting channel are determined by two attenuation lengths, the length \( L^{(f)}_n \) of forward scattering and the backscattering length \( L^{(b)}_n \),

\[
\frac{1}{L^{(f)}_n} = \sigma^2 \frac{(\pi n / d)^2}{k_n d} \sum_{n'=1}^{N_d} \frac{(\pi n' / d)^2}{k_{n'} d} W(k_n - k_{n'}),
\]

(2.2)

\[
\frac{1}{L^{(b)}_n} = \sigma^2 \frac{(\pi n / d)^2}{k_n d} \sum_{n'=1}^{N_d} \frac{(\pi n' / d)^2}{k_{n'} d} W(k_n + k_{n'}).
\]

(2.3)

These scattering lengths depend on the root-mean-square roughness height \( \sigma \), the lengthwise wave number \( k_n = \sqrt{k^2 - (\pi n / d)^2} \) of \( n \)-th propagating mode and (what is important) on the roughness power spectrum \( W(k_x) \) that is the Fourier transform of the binary correlator of the surface profile.

From Eqs. (2.2) and (2.3), one can see that in general both attenuation lengths are contributed by scattering of a given \( n \)-th mode into all others. This is the case when, for example, surface profile is either of white-noise type with constant power spectrum, or has fast decaying correlations described by a widely used Gaussian correlator (or, the same, with a slow decrease of its Fourier transform). Besides, these expressions manifest rather strong dependence on the mode index \( n \). Specifically, the larger the mode number \( n \), the smaller the corresponding attenuation lengths and as a consequence, the stronger the scattering of this mode into the others. As was shown in Refs. [7,8] an interesting phenomenon of the coexistence of ballistic, diffusive, and localized transport arises, which seems to be generic for waveguides with a surface disorder. Namely, while lowest modes can be in the ballistic regime, the intermediate and highest modes exhibit the diffusive and localized behavior, respectively. As a result, the hierarchy of mode attenuation lengths emerges. One can also see that due to inter-mode transitions all the conducting channels turn out to be mixed and therefore, the waveguide of large enough length is non-transparent.

Now let us demonstrate that the situation fundamentally changes when the surface roughness has specific long-range correlations. As an example, we consider the random surface with the simplest power spectrum in the form of a “window function”,

\[
W(k_x) = (\pi / k_c) \Theta(k_c - |k_x|).
\]

(2.4)

Here \( \Theta(\varepsilon) \) stands for the unit-step function and the characteristic wave number \( k_c > 0 \) is the correlation parameter to be specified. Note that the rough surfaces with predefined correlations can be fabricated with the use of the methods developed in Ref. [9]. Specifically, having a desirable form of \( W(k_x) \) one can obtain the surface profile \( \xi(x) \) as a convolution of a white noise \( Z(x) \) with the function whose Fourier transform is \( W^{1/2}(k_x) \). In such a way rough surfaces with the power spectrum (2.4) can be constructed as,

\[
\xi(x) = \frac{\sigma}{\sqrt{\pi k_c}} \int_{-\infty}^{\infty} dx' Z(x - x') \frac{\sin(k_c x')}{x'},
\]

(2.5)

where the \( x \) axis stretches along the waveguide. Note also that surfaces with rectangular spectrum are not exotic. They have been recently fabricated in experimental study of enhanced backscattering [10].

In the case of the roughness spectrum (2.4) the number of modes into which a given \( n \)-th mode is scattered, i.e. the actual number of summands in Eqs. (2.2) and (2.3), is determined by the spectral width \( k_c \). Moreover, if the distance between neighboring wave numbers is larger than \( k_c \),

\[
|k_n - k_{n\pm 1}| > k_c,
\]

(2.6)
then the transitions between all propagating modes are forbidden. As a consequence, the sum over \( n' \) in Eq. (2.2) contains only one term with \( n' = n \) which describes the direct intra-mode scattering only. At the same time, each term in the sum (2.3) is equal to zero so that the backscattering length diverges. So, we obtain the finite value for the forward scattering length and infinite backscattering length,

\[
I_n^{(f)} = \left( k_c / \pi \sigma \right)^2 (k_n d)^2 / (\pi n d)^4.
\]

(2.7)

The above consideration gives rise to remarkable conclusions. Specifically: (i) All the propagating modes that satisfy condition (2.6), are fully independent of other waveguide modes in spite of their interaction with a rough surface. In other words, they represent a coset of 1D non-interacting channels. (ii) As is well known from the standard theory of 1D localization, the transmission through any 1D disordered structure is determined by the backscattering length only and does not depend on the forward scattering. Since the former is infinite for every independent channel, all of them are completely transparent with partial transmittances \( T_n = 1 \). (iii) As for other propagating modes with index \( n \) in contradiction with the requirement (2.6), they remain to be mixed by surface scattering and non-transparent for long enough waveguides. (iv) It is evidently that mixed non-transparent channels do not contribute to the total transmittance (2.1). Consequently, the latter turns out to be equal to the total number of independent transparent modes. (v) Inequality (2.6) restricts the mode index \( n \) from below. Therefore, in contrast to the conventional situation, the low modes are mixed and non-transparent while the high propagating modes are independent and ballistic. Because of the abrupt behaviour of roughness power spectrum (2.4), the transition from mixed to independent modes is also abrupt.

3. Step-wise transmittance

From Eq. (2.6) with large number of conducting channels, \( N_d = [kd / \pi] = kd / \pi >> 1 \), one can easily obtain the number of mixed non-transparent modes. Then the explicit expression for the waveguide transmittance is written as a difference between \( N_d \) and the number of mixed modes,

\[
<T> = [kd / \pi] - [(kd / \pi)] / \alpha_c,
\]

(3.1)

\( \alpha_c = \sqrt{1 + (k_c d / \pi)^2} \).

The presented figure shows the dependence of \( <T> \) on the mode parameter \( kd / \pi \) for the value \( k_c d / \pi = 0.32 \). One can see an unusual step-wise dependence for some values of the control parameter \( \alpha_c \) that is defined by the width \( k_c \) of the rectangular power spectrum (2.4).

Within the region \( kd / \pi < \alpha_c \), the second term in Eq. (3.1) is zero and all propagating modes are independent and transparent. Here the transmittance exhibits a ballistic step-wise increase. Each step up arises for an integer value of \( kd / \pi \), when a new conducting channel emerges.
Otherwise, when $kd/\pi > \alpha_c$, in addition to the standard steps up due to the first term of Eq. (3.1) there are also the steps down due to the second term. These steps down are formed by the correlated surface scattering and arise when a successive low mode becomes mixed and non-transparent. The positions of the steps down are at integer values of $kd/\pi \alpha$. Since these values are determined by the correlation parameter $k_c$, in general, they do not coincide with the integer values of $kd/\pi$.

Evidently, there may be a situation when the steps up and down cancel each other within some interval of the mode parameter $kd/\pi$. Thus, the experimental observation of the discussed dependence seems to be highly interesting.

4. Summary

We have considered transport properties of many-mode plane waveguides (or quasi-1D conducting wires) with correlated surface disorder. It was shown that the effect of long-range correlations is much more sophisticated in comparison with that for single-mode waveguides. First, the long-range correlations give rise to a suppression of the interaction between different propagating modes. This non-trivial action turns out to be crucial for the reduction of a system of mixed channels with quasi-1D (diffusive or localized) transport, to the coset of independent waveguide modes with purely 1D transport. Second, the same correlations provide a perfect transparency of each independent channel, similar to what happens in 1D geometry.

The number of independent transparent modes is governed by the correlation parameter and can be equal to the total number of conducting channels. Therefore, the total transmission can be significantly enhanced in comparison with the case of uncorrelated surface roughness.

The total waveguide transmittance exhibits a step-wise dependence on the mode parameter $kd/\pi$ with both steps up and steps down. Each step up arises when the value of $kd/\pi$ increases by one and new propagating normal mode emerges. The steps down are formed by the correlated surface scattering, and arise at fractional values of $kd/\pi$ where a successive lower conducting channel becomes mixed and non-transparent.

We have also shown how to construct rough surfaces resulting in a perfect transparency within any given frequency window of an incoming wave.

Our study may find practical applications when fabricating of waveguides and electron/optic devices with a selective transport.

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