



# Tunable Acoustic Superscattering for Subwavelength Structure

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## BACKGROUND

Superscattering, which greatly enhances the interaction between waves and matter, plays a crucial role in wave manipulation across acoustics and optics. The scattering intensity of superscattering significantly exceeds the conventional limit, characterized by the far-field spreading pattern in field distribution.

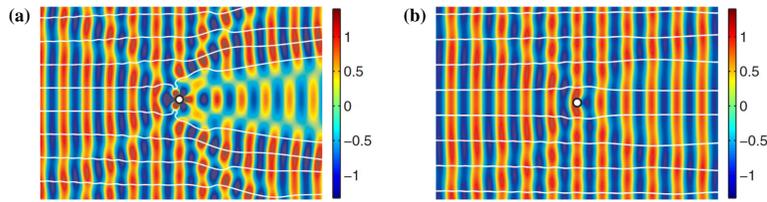


FIG 1. Field distribution of (a) superscattering<sup>[1]</sup> and (b) normal scattering.

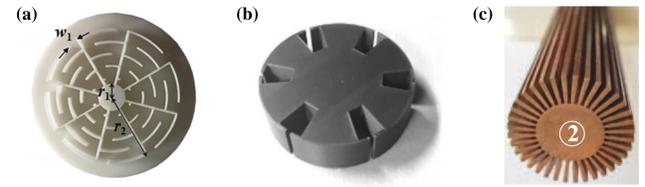


FIG 2. Figures of (a) coiling-up space<sup>[2]</sup>, (b) Helmholtz resonator<sup>[3]</sup> and (c) corrugated cylinder<sup>[4]</sup>.

In acoustic superscattering, the main challenge is the complex design and limited tunability of the superscatterer, which constrain the phenomenon to narrow frequency bands. Therefore, the tunable superscattering which could broaden the operational frequency range is needed for diverse applications.

## ANISOTROPIC MODEL

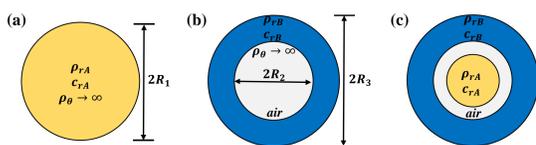


FIG 3. Schematic of (a) circular, (b) annular and (c) composite scatterer.

The materials are characterized by radial sound speed  $c_r$ , radial mass density  $\rho_r$ , and azimuthal mass density  $\rho_\theta$ . The impedance  $\sqrt{c_r \rho_r}$  of each scatterer is matched to the surrounding air, ensuring efficient wave coupling and enhanced acoustic scattering.

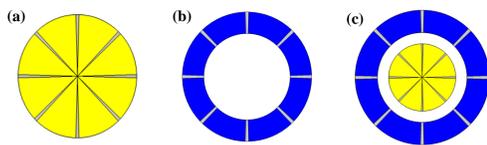


FIG 4. Realization of anisotropy for (a) circular, (b) annular and (c) composite scatterer.

The anisotropy is realized by hard wall slits separating the material, which results in extreme azimuthal mass density.

## SCATTERING CHARACTERISTIC

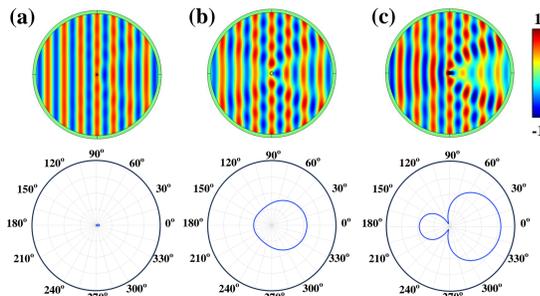


FIG 5. Pressure field and far-field distribution of models in FIG 4 with  $R_1=7.5\text{cm}$ ,  $R_2=10\text{cm}$  and  $R_3=15\text{cm}$  at  $f=268\text{ Hz}$  with  $n_A=4$  and  $n_B=5$ .

Across the pressure field distributions, the composite scatterer exhibits significantly enhanced scattering compared to individual circular or annular scatterers. The superscattering is valid for the composite structure.

The far-field radiation pattern of composite scatterer highlights the **directional scattering intensity**, compared with the weak scattering of circular scatterer and the uniform scattering of annular scatterer.

## TUNABLE SUPERSCATTERING

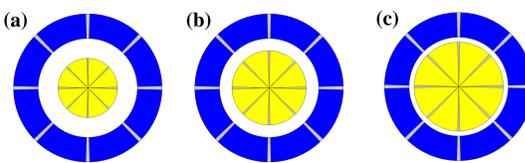


FIG 6. Tunable Superscatterers with air thicknesses of (a)  $t=4\text{ cm}$ , (b)  $t=2.5\text{ cm}$  and (c)  $t=1\text{ cm}$  with  $n_A=4$  and  $n_B=5$ .

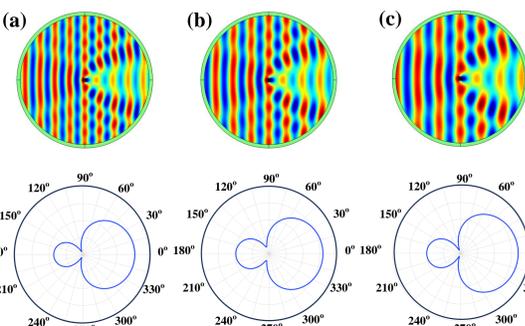


FIG 7. Pressure field and far-field distribution of (a)  $t=4\text{ cm}$ ,  $f=314\text{ Hz}$ , (b)  $t=2.5\text{ cm}$ ,  $f=265\text{ Hz}$  and (c)  $t=1\text{ cm}$ ,  $f=221\text{ Hz}$ .

The tunable superscattering is achieved by changing the air layer thickness. The reduced speed  $c_{rA}$  of the circular scatterer inside the cavity, which is significantly lower than the background air speed  $c_0$ , **enables the superscattering to occur at low frequencies**.

The pressure field distribution and far-field pattern of tunable scatterers all show the superscattering phenomenon, which **validates the tunability and effectiveness** of the design.

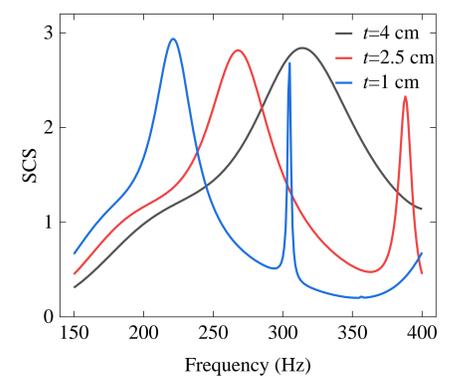


FIG 8. Normalized total SCSs calculated from different thicknesses  $t$  of all the composite scatterers above.

In FIG 8, superscattering can be obtained across a broad frequency range. Results indicate that thinner air layers lead to lower resonance frequencies, enabling precise control over scattering behavior for specific application.

## SUPERSCATTERING MECHANISM

We use **Mie scattering theory** in anisotropic media to analyze the model<sup>[5]</sup>. The general expression for the pressure field is:

$$p(r, \theta) = \sum_{m=-\infty}^{\infty} [a_m J_m(k_r r) + b_m H_m^{(1)}(k_r r)] e^{im\theta},$$

where  $v = m\sqrt{\rho_r/\rho_\theta}$ , which related to anisotropy. In extreme anisotropic medium, the high-order terms are suppressed, causing  $v \rightarrow 0$ , which significantly alters the scattering behavior.

Consider an anisotropic circular scatterer under plane wave incidence, we express the pressure field inside and outside the scatterer respectively:

$$p_1(r, \theta) = \sum_{m=-\infty}^{\infty} A_m J_0(k_r r) e^{im\theta},$$

$$p_2(r, \theta) = \sum_{m=-\infty}^{\infty} [i^m J_m(k_0 r) + D_m H_m^{(1)}(k_0 r)] e^{im\theta},$$

Applying the boundary conditions for pressure and velocity continuity, we derive the scattering coefficients :

$$D_m = \frac{J_m(k_0 R_1) J_0'(k_r R_1) - J_m'(k_0 R_1) J_0(k_r R_1)}{H_m^{(1)'}(k_0 R_1) J_0(k_r R_1) - H_m^{(1)}(k_0 R_1) J_0'(k_r R_1)},$$

**the anisotropy forces multipole modes to degenerate at lower frequencies**, leading to constructive interference and thus total scattering cross-section  $D = \frac{4}{k_0} \sum_{m=-\infty}^{\infty} |D_m|^2$  exceeds the single-channel limit.

## CONCLUSION

Main Contributions:

- Tunable acoustic superscatterer
- Anisotropic Mie scattering theory
- Dynamic broadband superscattering

## OUTLOOK

Possible Applications:

- Ultrasound diagnostics: Improved imaging resolution
- Underwater communication: Enhanced frequency range
- Architectural acoustics: Optimized sound control

## ACKNOWLEDGEMENT

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